Package 'CircNNTSR'

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Type Package

Title Statistical Analysis of Circular Data using Nonnegative Trigonometric Sums (NNTS) Models

Version 2.3

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Description Includes functions for the analysis of circular data using distributions based on Nonnegative Trigonometric Sums (NNTS). The package includes functions for calculation of densities and distributions, for the estimation of parameters, for plotting and more.

Depends stats

License GPL (>= 2)

LazyLoad yes

NeedsCompilation no

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R topics documented:

`CircNNTSR-package` *CircNNTSR: An R Package for the statistical analysis of circular data using nonnegative trigonometric sums (NNTS) models*

Description

A collection of utilities for the statistical analysis of circular and spherical data using nonnegative trigonometric sum (NNTS) models

Details

Package:	CircNNTSR
Type:	Package
Version:	2.2-1
Date:	2020-02-16
License:	GLP (>=2)
LazyLoad:	yes

Fernandez-Duran, J.J. (2004) proposed a new family of distributions for circular random variables based on nonnegative trigonometric sums. This package provides functions for working with circular distributions based on nonnegative trigonometric sums, including functions for estimating the parameters and plotting the densities.

The distribution function in this package is a circular distribution based on nonnegative trigonometric sums (Fernandez-Duran, 2004). Fejer (1915) expressed a univariate nonnegative trigonometric (Fourier) sum (series), for a variable θ, as the squared modulus of a sum of complex numbers, i.e.,

$$\left\|\sum_{k=0}^{M} c_k e^{ik\theta}\right\|^2 \quad (1)$$

where $i = \sqrt{-1}$. From this result, the parameters (a_k, b_k) for $k = 1, \ldots, M$ of the trigonometric sum of order M,$T(\theta)$,

$$T(\theta) = a_0 + \sum_{k=1}^{M}(a_k cos(k\theta) + b_k sin(k\theta))$$

are expressed in terms of the complex parameters in Equation 1 , c_k, for $k = 0, \ldots, M$, as $a_k - ib_k = 2\sum_{\nu=0}^{n-k} c_{\nu+k}\bar{c}_\nu$. The additional constraint, $\sum_{k=0}^{n} \|c_k\|^2 = \frac{1}{2\pi} = a_0$, is imposed to make the trigonometric sum to integrate one. Thus, c_0 must be real and positive, and there are 2*M free parameters. Then, the probability density function for a circular (angular) random variable is defined as (Fernandez-Duran, 2004)

$$f(\theta; \underline{a}, \underline{b}, M) = \frac{1}{2\pi} + \frac{1}{\pi}\sum_{k=1}^{M}(a_k cos(k\theta) + b_k sin(k\theta)).$$

Note that Equation 1 can also be expressed as a double sum as

$$\sum_{k=0}^{M}\sum_{m=0}^{M} c_k \bar{c}_m e^{i(k-m)\theta}$$

.

The $\underline{c}$ parameters can also be expressed in polar coordinates as $c_k = \rho_k e^{i\phi_k}$ for $\rho_k \geq 0$ and $\phi_k \in [0, 2\pi)$; where ρ_k is the modulus of c_k and ϕ_k is the argument of c_k for $k = 1, \ldots, M$. Many functions of the packages use as parameters the squared moduli and the arguments of c_k, ρ_k^2 and ϕ_k, for $k = 1, \ldots, M$. We refer to the parameter M as the number of components in the NNTS.

Author(s)

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

Maintainer: Maria Mercedes Gregorio Dominguez <mercedes@itam.mx>

References

Fernandez-Duran, J.J. (2004). Circular Distributions Based on Nonnegative Trigonometric Sums, Biometrics, 60(2), 499-503.

Fernandez-Duran, J.J., Gregorio-Dominguez, M.M. (2010). A Likelihood-Ratio Test for Homogeneity in Circular Data. Journal of Biometrics & Biostatistics, 1(3), 107. doi:10.4172/2155-6180.1000107

Fernandez-Duran, J.J., Gregorio-Dominguez, M.M. (2010). Maximum Likelihood Estimation of Nonnegative Trigonometric Sums Models Using a Newton-Like Algorithm on Manifolds. Electronic Journal of Statistics, 4, 1402-1410. doi:10.1214/10-ejs587

Fernandez-Duran, J.J., Gregorio-Dominguez, M.M. (2014). Distributions for Spherical Data Based on Nonnegative Trigonometric Sums. Statistical Papers, 55(4), 983-1000. doi:10.1007/s00362-013-0547-5

Fernandez-Duran, J.J., Gregorio-Dominguez, M.M. (2014). Modeling Angles in Proteins and Circular Genomes Using Multivariate Angular Distributions Based on Nonnegative Trigonometric Sums. Statistical Applications in Genetics and Molecular Biology, 13(1), 1-18. doi:10.1515/sagmb-2012-0012

Fernandez-Duran, J.J., Gregorio-Dominguez, M.M. (2014). Testing for Seasonality Using Circular Distributions Based on Nonnegative Trigonometric Sums as Alternative Hypotheses. Statistical Methods in Medical Research, 23(3), 279-292. doi:10.1177/0962280211411531.

Juan Jose Fernandez-Duran, Maria Mercedes Gregorio-Dominguez (2016). CircNNTSR: An R Package for the Statistical Analysis of Circular, Multivariate Circular, and Spherical Data Using Nonnegative Trigonometric Sums. Journal of Statistical Software, 70(6), 1-19. doi:10.18637/jss.v070.i06

Examples

```
set.seed(200)
data(Turtles_radians)
#Empirical analysis of data
Turtles_hist<-hist(Turtles_radians,breaks=10,freq=FALSE)
#Estimation of the NNTS density with 3 componentes for data
```

```
est<-nntsmanifoldnewtonestimation(Turtles_radians,3,iter=100)
est
#plot the estimated density
nntsplot(est$cestimates[,2],3)
#add the histogram to the estimated density plot
plot(Turtles_hist, freq=FALSE, add=TRUE)

b<-c(runif(10,3*pi/2,2*pi-0.00000001),runif(10,pi/2,pi-0.00000001))
estS<-nntsestimationSymmetric(2,b)
nntsplotSymmetric(estS$coef,2)

M<-c(2,3)
R<-length(M)
data(Nest)
data<-Nest
est<-mnntsmanifoldnewtonestimation(data,M,R,100)
est
cest<-est$cestimates
mnntsplot(cest, M)
```

Ants *Movements of ants*

Description

Directions chosen by 100 ants in response to an evenly illuminated black target.

Usage

```
data(Ants)
```

Format

Directions chosen by 100 ants in degrees

Source

Randomly selected values by Fisher (1993) from Jander (1957)

References

N.I. Fisher (1993) Statistical analysis of circular data. Cambridge University Press.

`Ants_radians` *Movements of ants*

Description

Direction chosen by 100 ants in response to an evenly illuminated black target.

Usage

```
data(Ants_radians)
```

Format

Directions chosen by 100 ants in radians

Source

Randomly selected values by Fisher (1993) from Jander (1957)

References

N.I. Fisher (1993) Statistical analysis of circular data. Cambridge University Press.

`Datab3fisher` *Database B3 from Fisher*

Description

Database B3 from Fisher et al. (1987)

Usage

```
data(Datab3fisher)
```

Format

Datab3fisher

Details

The dataset Datab3fisher consists of 148 observations of the arrival directions of low-mu showers of cosmic rays (Toyoda *et al.*, 1965; see Fisher *et al.*, 1987, pp. 280-281). The observations are measured in declination and right ascension coordinates.

Source

Fisher, et al. (1987)

References

Toyoda, Y., Suga, K., Murakami, K., Hasegawa, H., Shibata, S., Domingo, V., Escobar, I., Kamata, K., Bradt, H., Clark, G. and La Pointe, M. (1965). Studies of Primary Cosmic Rays in the Energy Region 10^{14} eV to 10^{17} eV (Bolivian Air Shower Joint Experiment), Proceedings of the International Conference on Cosmic Rays, vol. 2, London, September, 1965, 708–711. London: The Institute of Physics and the Physical Society.

Fisher, N.I., Lewis, T. and Embleton, B.J.J. (1987). Statistical Analysis of Spherical Data, Cambridge U.K.: Cambridge University Press.

Datab3fisher_ready *Data transformed from Datab3fisher*

Description

Data transformed from Datab3fisher

Usage

```
data(Datab3fisher_ready)
```

Format

Datab3fisher_ready

Details

datab3fisher[,2] <- 90 + datab3fisher[,2]; datab3fisher_ready <- datab3fisher*(pi/180)

DataB5FisherSpherical *Spherical Data on Magnetic Remanence*

Description

Measurements of magnetic remanence from 52 specimens of red beds from the Bowen Basin, Queensland.

Usage

```
data(DataB5FisherSpherical)
```

Format

Declination -inclination in degrees

Source

P.W. Schmidt

References

Fisher N.I., Lewis T. and Embleton B.J.J. (1987) Statistical Analysis of Spherical Data. Cambridge University Press, Cambridge. Data B.5.

Examples

```
data(DataB5FisherSpherical)
```

`Datab6fisher` *Database B6 from Fisher et al. (1987)*

Description

datab6fisher

Usage

```
data(Datab6fisher)
```

Format

The coordinates are declination and inclination measured in degrees

Details

The data-set Datab6fisher contains 107 measurements of magnetic remanence in samples of Precambrian volcanics collected in Northwest Australia. (Schmidt and Embleton, 1985; see Fisher et al., 1987, pp. 285).

Source

Fisher, et al. (1987)

References

Schmidt, P.W. and Embleton, B.J.J. (1985). Pre-folding and overprint magnetic signatures in Precambrian (~2.9-2.7ga) igneous rocks from the Pilbara Craton and Hamersley Basin, N.W. Australia, Journal of Geophysical Research, 90 (B4), 2967–2984.

Fisher, N.I., Lewis, T. and Embleton, B.J.J. (1987). Statistical Analysis of Spherical Data, Cambridge U.K.: Cambridge University Press.

Datab6fisher_ready   *Data transformed from datab6fisher*

Description

Data transformed from datab6fisher

Usage

```
data(Datab6fisher_ready)
```

Format

Datab6fisher_ready

Details

dataaux <- datab6fisher; datab6fisher[,1] <- dataaux[,2]; datab6fisher[,2] <- dataaux[,1]; datab6fisher[,1] <- 360 - datab6fisher[,1]; datab6fisher[,2] <- 90 + datab6fisher[,2]; datab6fisher_ready <- datab6fisher*(pi/180)

DataUniformBivariate200obs   *Uniform Bivariate Circular Data*

Description

200 realizations of a uniform distribution on the torus

Usage

```
data(DataUniformBivariate200obs)
```

Format

Angles in radians

`EarthquakesPacificMexicogt6` *Date of Occurrence of Earthquakes*

Description

The time of occurrence of earthquakes of intensity greater than 6.0^o Richter with an epicenter occurring in the coast of the Pacific Ocean in Mexico from 1920 to 2002. There is a total of 241 observations.

Usage

```
data(EarthquakesPacificMexicogt6)
```

Format

Time in years. All observations in the interval (0,1]

Source

Mexican Database of Strong Earthquakes. CENAPRED.

`EarthquakesPacificMexicogt7` *Date of Occurrence of Earthquakes 2*

Description

The time of occurrence of earthquakes of intensity greater than 7.0^o Richter with an epicenter occurring in the coast of the Pacific Ocean in Mexico from 1920 to 2002. There are a total of 76 observations.

Usage

```
data(EarthquakesPacificMexicogt7)
```

Format

Time in years. All observations in the interval (0,1]

Source

Mexican Database of Strong Earthquakes. CENAPRED.

`HomicidesMexico2005` *Homicides in Mexico during 2005*

Description

Monthly number of homicides in Mexico during 2005

Usage

```
data(HomicidesMexico2005)
```

Format

Integer values

Source

INEGI (Mexican National Statistical Agency) www.inegi.gob.mx

`HurricanesGulfofMexico1951to1970` *Hurricanes in Mexico from 1951 to 1970*

Description

The time of occurrence (starting times) of hurricanes in the Gulf of Mexico for the 1951-1970 period. There are a total of 196 observations.

Usage

```
data(HurricanesGulfofMexico1951to1970)
```

Format

Time in years. All observations in the interval (0,1]

Source

http://weather.unisys.com/hurricane/atlantic/1978/index.html

`HurricanesGulfofMexico1971to2008`
Hurricanes in Mexico from 1971 to 2008

Description

The time of occurrence (starting times) of hurricanes in the Gulf of Mexico for the 1971-2008 period. There are a total of 417 observations

Usage

```
data(HurricanesGulfofMexico1971to2008)
```

Format

Time in years. All observations in the interval (0,1]

Source

http://weather.unisys.com/hurricane/atlantic/1978/index.html

`mnntsdensity` *MNNTS density function*

Description

Density function for the MNNTS model

Usage

```
mnntsdensity(data, cpars = 1/sqrt(2 * pi), M = 0, R=1)
```

Arguments

`data`	Matrix of angles in radians, a column for each dimension, a row for each data point
`cpars`	Parameters of the model. A vector of complex numbers of dimension prod(M+1). The first element is a real and positive number. The first M[1]+1 elements correspond to dimension one, the next M[2]+1 elements correspond to dimension two, and so on. The sum of the SQUARED moduli of the c parameters must be equal to $\left(\frac{1}{2*pi}\right)^R$.
`M`	Vector of length R with number of components in the MNNTS for each dimension
`R`	Number of dimensions

Value

The function returns the density function evaluated at each row in data

Author(s)

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

References

Juan Jose Fernandez-Duran, Maria Mercedes Gregorio-Dominguez (2016). CircNNTSR: An R Package for the Statistical Analysis of Circular, Multivariate Circular, and Spherical Data Using Nonnegative Trigonometric Sums. Journal of Statistical Software, 70(6), 1-19. doi:10.18637/jss.v070.i06

Examples

```
set.seed(200)
M<-c(2,3)
R<-length(M)
data<-c(0,pi,pi/2,pi,pi,3*pi/2,pi,2*pi,2*pi,pi)
data<-matrix(data,ncol=2,byrow=TRUE)
data
ccoef<-mnntsrandominitial(M,R)
mnntsdensity(data,ccoef,M,R)

M<-c(2,3)
R<-length(M)
data(Nest)
data<-Nest
est<-mnntsmanifoldnewtonestimation(data,M,R,100)
est
ccoef<-est$cestimates[,3]
mnntsdensity(data,ccoef,M,R)
```

---

mnntsloglik *MNNTS log-likelihood function*

Description

Computes the log-likelihood function with MNNTS density for data

Usage

```
mnntsloglik(data, cpars = 1/sqrt(2 * pi), M = 0, R = 1)
```

Arguments

- `data` Matrix of angles in radians, a column for each dimension, a row for each data point.
- `cpars` Parameters of the model. A vector of complex numbers of dimension prod(M+1). The first element is a real and positive number. The first M[1]+1 elements correspond to dimension one, next M[2]+1 elements correspond to dimension two, and so on. The sum of the SQUARED moduli of the c parameters must be equal to $\left(\frac{1}{2*pi}\right)^R$.
- `M` Vector of length R with number of components in the MNNTS for each dimension.
- `R` Number of dimensions.

Value

The function returns the value of the log-likelihood function for the data.

Author(s)

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

References

Fernandez-Duran, J.J. and Gregorio-Dominguez, M.M. (2009) Multivariate Angular Distributions Based on Multiple Nonnegative Trigonometric Sums, Working Paper, Statistics Department, ITAM, DE-C09.1

Examples

```
M<-c(2,3)
R<-length(M)
data<-c(0,pi,pi/2,pi,pi,3*pi/2,pi,2*pi,2*pi,pi)
data<-matrix(data,ncol=2,byrow=TRUE)
data
ccoef<-mnntsrandominitial(M,R)
mnntsdensity(data,ccoef,M,R)
mnntsloglik(data,ccoef,M,R)
```

---

mnntsmanifoldnewtonestimation

Parameter estimation for the MNNTS distributions

Description

Computes the maximum likelihood estimates of the MNNTS parameters using a Newton algorithm on the hypersphere

Usage

```
mnntsmanifoldnewtonestimation(data,M=0,R=1,iter=1000,initialpoint=FALSE,cinitial)
```

Arguments

`data`	Matrix of angles in radians, a column for each dimension, a row for each data point
`M`	Vector of length R with number of components in the MNNTS for each dimension
`R`	Number of dimensions
`iter`	Number of iterations for the Newton algorithm
`initialpoint`	TRUE if an initial point for the optimization algorithm will be used
`cinitial`	Initial value for cpars (parameters of the model) for the optimization algorithm. Vector of complex numbers of dimension prod(M+1). The first element is a real and positive number. The first M[1]+1 elements correspond to dimension one, the next M[2]+1 elements correspond to dimension two, and so on. The sum of the SQUARED moduli of the c parameters must be equal to $\left(\frac{1}{2*pi}\right)^R$.

Value

`cestimates`	Matrix of prod(M+1)*(R+1). The first R columns are the parameter number, and the last column is the c parameter's estimators
`loglik`	Optimum log-likelihood value
`AIC`	Value of Akaike's Information Criterion
`BIC`	Value of Bayesian Information Criterion
`gradnormerror`	Gradient error after the last iteration

Author(s)

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

References

Fernandez-Duran, J.J. and Gregorio-Dominguez, M.M. (2009) Multivariate Angular Distributions Based on Multiple Nonnegative Trigonometric Sums, Working Paper, Statistics Department, ITAM, DE-C09.1

Fernandez-Duran, J.J., Gregorio-Dominguez, M.M. (2014). Modeling Angles in Proteins and Circular Genomes Using Multivariate Angular Distributions Based on Nonnegative Trigonometric Sums. Statistical Applications in Genetics and Molecular Biology, 13(1), 1-18. doi:10.1515/sagmb-2012-0012

Examples

```
set.seed(200)
M<-c(2,3)
R<-length(M)
data(Nest)
data<-Nest*(pi/180)
est<-mnntsmanifoldnewtonestimation(data,M,R,100)
est
```

`mnntsmarginal` *Marginal density function of the MNNTS model*

Description

Marginal density function for one dimension of the MNNTS model evaluated at a point

Usage

```
mnntsmarginal(cestimatesarray, M, component, theta)
```

Arguments

- `cestimatesarray` Matrix of prod(M+1)*(R+1). The first R columns are the parameter number, and the last column is the c parameter's estimators
- `M` Vector of length R with number of components in the MNNTS for each dimension
- `component` Number of the dimension for computing the marginal
- `theta` An angle in radians (or a vector of angles)

Value

The function returns the density function evaluated at theta

Author(s)

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

References

Fernandez-Duran, J.J. and Gregorio-Dominguez, M.M. (2009) Multivariate Angular Distributions Based on Multiple Nonnegative Trigonometric Sums, Working Paper, Statistics Department, ITAM, DE-C09.1

Fernandez-Duran, J.J., Gregorio-Dominguez, M.M. (2014). Modeling Angles in Proteins and Circular Genomes Using Multivariate Angular Distributions Based on Nonnegative Trigonometric Sums. Statistical Applications in Genetics and Molecular Biology, 13(1), 1-18. doi:10.1515/sagmb-2012-0012

Examples

```
set.seed(200)
M<-c(2,3)
R<-length(M)
data(Nest)
data<-Nest
est<-mnntsmanifoldnewtonestimation(data,M,R,100)
est
cest<-est$cestimates
mnntsmarginal(cest,M,1,pi)
```

---

mnntsplot *Plots an MNNTS bivariate density*

Description

Plots the MNNTS bivariate density function

Usage

```
mnntsplot(cestimates, M, ...)
```

Arguments

`cestimates`	Matrix of prod(M+1)*(R+1). The first R columns are the parameter number, and the last column is the c parameter's estimators. R=2 for a bivariate distribution
`M`	Vector with the number of components in the MNNTS for each dimension
`...`	Arguments passed to the function plot

Author(s)

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

References

Fernandez-Duran, J.J. and Gregorio-Dominguez, M.M. (2009) Multivariate Angular Distributions Based on Multiple Nonnegative Trigonometric Sums, Working Paper, Statistics Department, ITAM, DE-C09.1

Fernandez-Duran, J.J., Gregorio-Dominguez, M.M. (2014). Modeling Angles in Proteins and Circular Genomes Using Multivariate Angular Distributions Based on Nonnegative Trigonometric Sums. Statistical Applications in Genetics and Molecular Biology, 13(1), 1-18. doi:10.1515/sagmb-2012-0012

Examples

```
set.seed(200)
M<-c(2,3)
R<-length(M)
data(Nest)
data<-Nest
est<-mnntsmanifoldnewtonestimation(data,M,R,100)
est
cest<-est$cestimates
mnntsplot(cest, M)
```

---

mnntsplotmarginal *Plots an MNNTS marginal density*

Description

Plots the MNNTS marginal density function

Usage

```
mnntsplotmarginal(cestimates, M, component, ...)
```

Arguments

cestimates	Matrix of prod(M+1)*(R+1). The first R columns are the parameter number, and the last column the c parameter's estimators. The matrix could be the output of mnntsmanifoldnewtonestimation $cestimates
M	Vector with number of components in the MNNTS for each dimension
component	Number of the dimension for computing the marginal density
...	Arguments passed to the function plot

Author(s)

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

References

Fernandez-Duran, J.J. and Gregorio-Dominguez, M.M. (2009) Multivariate Angular Distributions Based on Multiple Nonnegative Trigonometric Sums, Working Paper, Statistics Department, ITAM, DE-C09.1

Fernandez-Duran, J.J., Gregorio-Dominguez, M.M. (2014). Modeling Angles in Proteins and Circular Genomes Using Multivariate Angular Distributions Based on Nonnegative Trigonometric Sums. Statistical Applications in Genetics and Molecular Biology, 13(1), 1-18. doi:10.1515/sagmb-2012-0012

Examples

```
set.seed(200)
M<-c(2,3)
R<-length(M)
data(Nest)
data<-Nest
est<-mnntsmanifoldnewtonestimation(data,M,R,100)
est
cest<-est$cestimates
mnntsplotmarginal(cest, M, 1)
mnntsplotmarginal(cest, M, 2)
```

---

mnntsplotwithmarginals

Plots an MNNTS bivariate density together with the marginals

Description

Plots the MNNTS bivariate density function together with the marginals

Usage

```
mnntsplotwithmarginals(cestimates, M, ...)
```

Arguments

`cestimates`	Matrix of prod(M+1)*(R+1). The first R columns are the parameter number, and the last column the c parameter's estimators. The matrix could be the output of mnntsmanifoldnewtonestimation $cestimates.
`M`	Vector of length R with number of components in the MNNTS for each dimension
`...`	Arguments passed to the function plot

Author(s)

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

References

Fernandez-Duran, J.J. and Gregorio-Dominguez, M.M. (2009) Multivariate Angular Distributions Based on Multiple Nonnegative Trigonometric Sums, Working Paper, Statistics Department, ITAM, DE-C09.1

Fernandez-Duran, J.J., Gregorio-Dominguez, M.M. (2014). Modeling Angles in Proteins and Circular Genomes Using Multivariate Angular Distributions Based on Nonnegative Trigonometric Sums. Statistical Applications in Genetics and Molecular Biology, 13(1), 1-18. doi:10.1515/sagmb-2012-0012

Examples

```
set.seed(200)
M<-c(2,3)
R<-length(M)
data(Nest)
data<-Nest
est<-mnntsmanifoldnewtonestimation(data,M,R,100)
est
cest<-est$cestimates
mnntsplotwithmarginals(cest, M)
```

---

mnntsrandominitial *Initial random point*

Description

This function generates a random point on the surface of the prod(M+1)-dimensional unit hypersphere

Usage

```
mnntsrandominitial(M = 1, R = 1)
```

Arguments

M	Vector of length R with number of components in the MNNTS for each dimension
R	Number of dimensions

Value

Returns a valid initial point for estimation functions

Author(s)

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

References

Fernandez-Duran, J.J. and Gregorio-Dominguez, M.M. (2009) Multivariate Angular Distributions Based on Multiple Nonnegative Trigonometric Sums, Working Paper, Statistics Department, ITAM, DE-C09.1

Examples

```
M<-c(2,3)
R<-length(M)
mnntsrandominitial(M,R)
```

mnntssimulation *MNNTS density simulation function*

Description

Simulation for the density function for the MNNTS model

Usage

```
mnntssimulation(nsim=1, cpars = 1/(2 * pi), M = c(0,0), R=2)
```

Arguments

nsim	Number of simulations
cpars	Parameters of the model. A vector of complex numbers of dimension prod(M+1). The first element is a real and positive number. The first M[1]+1 elements correspond to dimension one, next M[2]+1 elements correspond to dimension two, and so on. The sum of the SQUARED moduli of the c parameters must be equal to 1/(2*pi).
M	Vector of length R with number of components in the MNNTS for each dimension
R	Number of dimensions

Value

simulations	The function generates nsim random values from the MNNTS density function
conteo	Number of uniform random numbers used for simulations

Author(s)

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

References

Fernandez-Duran, J.J. and Gregorio-Dominguez, M.M. (2009) Multivariate Angular Distributions Based on Multiple Nonnegative Trigonometric Sums, Working Paper, Statistics Department, ITAM, DE-C09.1

Examples

```
M<-c(2,3)
R<-length(M)
ccoef<-mnntsrandominitial(M,R)
data<-mnntssimulation(10,ccoef,M,R)
data
```

Nest *Nest orientations and creek directions*

Description

Orientation of nests of 50 noisy scrub birds (theta) along the bank of a creek bed, together with the corresponding directions (phi) of creek flow at the nearest point to the nest.

Usage

```
data(Nest)
```

Format

Orientation of 50 nests (vectors)

Source

Data supplied by Dr. Graham Smith

References

N.I. Fisher (1993) Statistical analysis of circular data. Cambridge University Press.

nntsABcoefficients *AB coefficients*

Description

This function transforms the complex parameters c to the parameters ab for a reparameterization of the density function

Usage

```
nntsABcoefficients(cpars = 1/sqrt(2 * pi), M = 0)
```

Arguments

cpars	Vector of complex numbers of dimension M+1. The first element is a real and positive number. The sum of the SQUARED moduli of the c parameters must be equal to 1/(2*pi).
M	Number of components in the NNTS

Value

The function returns the parameters ab associated with the parameters cpars and returns a vector of real numbers of size 2*M, where the first M elements are the a_k, k=1,...,M, and the next M elements are the b_k, k=1,...,M

Author(s)

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

Examples

```
#random generation of c parameters
ccoef<-nntsrandominitial(3)
ccoef
ab<-nntsABcoefficients(ccoef,3)
ab
```

---

nntsABcoefficientsSymmetric

AB coefficients

Description

This function transforms the complex parameters c to the parameters ab for a reparameterization of the density function

Usage

```
nntsABcoefficientsSymmetric(cpars = c(0,0), M = 0)
```

Arguments

cpars	Vector of complex numbers of dimension 2*M
M	Number of components in the NNTS

Value

The function returns the parameters ab associated with the pararameters cpars

Author(s)

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

nntsABDensity *Density function with AB coefficients*

Description

Density function expressed in terms of the ab parameters at theta

Usage

```
nntsABDensity(theta, cpars = 1/sqrt(2 * pi), M = 0)
```

Arguments

theta	Vector of angles in radians
cpars	Vector of complex numbers of dimension M+1. The first element is a real and positive number. The sum of the SQUARED moduli of the c parameters must be equal to 1/(2*pi)
M	Number of components in the NNTS

Value

Returns the density function in terms of the ab coefficients evaluated at theta

Author(s)

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

Examples

```
ccoef<-nntsrandominitial(3)
nntsABDensity(1,ccoef,3)
nntsABDensity(1+2*pi,ccoef,3)
```

nntsABDensitySymmetric *Density function with AB coefficients*

Description

Density function expressed in terms of the ab parameters at theta

Usage

```
nntsABDensitySymmetric(cpars = c(0, 0), M = 0, theta)
```

Arguments

`theta`	Vector of angles in radians
`cpars`	Vector of complex numbers of dimension 2*M
`M`	Number of components in the NNTS

Value

Returns the density function in terms of the ab coefficients evaluated at theta

Author(s)

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

---

`nntsdensity` *NNTS density function*

Description

Density function for the NNTS model

Usage

```
nntsdensity(data, cpars = 1/sqrt(2 * pi), M = 0)
```

Arguments

`data`	Vector of angles in radians
`cpars`	Vector of complex numbers of dimension M+1. The first element is a real and positive number. The sum of the squared moduli of the c parameters must be equal to 1/(2*pi).
`M`	Number of components in the NNTS

Value

The function returns the density function evaluated at each point in data

Author(s)

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

References

Fernandez-Duran, J.J. (2004). Circular Distributions Based on Nonnegative Trigonometric Sums, Biometrics, 60(2), 499-503.

Juan Jose Fernandez-Duran, Maria Mercedes Gregorio-Dominguez (2016). CircNNTSR: An R Package for the Statistical Analysis of Circular, Multivariate Circular, and Spherical Data Using Nonnegative Trigonometric Sums. Journal of Statistical Software, 70(6), 1-19. doi:10.18637/jss.v070.i06

Examples

```
ccoef<-nntsrandominitial(3)
nntsdensity(1,ccoef,3)
nntsdensity(1+pi,ccoef,3)
nntsdensity(c(1,1+pi),ccoef,3)
```

---

nntsDensityInterval0to1 *NNTS density function for a variable defined in the interval [0,1)*

Description

Computes the density function at theta for a variable defined in the interval [0,1))

Usage

```
nntsDensityInterval0to1(S, cpars = 1/sqrt(2 * pi), M = 0)
```

Arguments

S	Vector of values defined in the interval [0,1) at which the density function is computed
cpars	Vector of complex numbers of dimension M+1. The first element is a real and positive number. The sum of the squared moduli of the c parameters must be equal to 1/(2*pi)
M	Number of components in the NNTS

Details

This function computes the density function of a variable S (S in the interval [0,1)). If theta is defined in radians (theta in the interval [0,2*pi)), the relation between S and theta is theta=2*pi*S.

Value

Value of the density function at each component of S

Author(s)

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

Examples

```
ccoef<-nntsrandominitial(3)
nntsDensityInterval0to1(c(.8,1.8),ccoef,3)
```

nntsDistribution *NNTS Distribution function*

Description

Cumulative distribution function in terms of the c parameters at theta, measured in radians [0,2*pi).

Usage

```
nntsDistribution(theta, cpars = 1/sqrt(2 * pi), M = 0)
```

Arguments

`theta`	Vector of angles in radians at which the distribution is computed
`cpars`	Vector of complex numbers of dimension M+1. The first element is a real and positive number. The sum of the SQUARED moduli of the c parameters must be equal to 1/(2*pi).
`M`	Number of components in the NNTS

Value

The function returns the value of the distribution function evaluated at each component of theta

Author(s)

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

Examples

```
ccoef<-nntsrandominitial(3)
nntsDistribution(c(0,pi/2,pi,2*pi-0.00000001,2*pi),ccoef,3)
```

nntsDistributioninterval0to1 *NNTS distribution function for the incidence data defined in the interval [0,1)*

Description

Computes the distribution function at theta for the incidence data (number of observed values in certain intervals defined in the interval [0,1))

Usage

```
nntsDistributioninterval0to1(theta, cpars = 1/sqrt(2 * pi), M = 0)
```

Arguments

theta	Value at which the distribution function is computed
cpars	Vector of complex numbers of dimension M+1. The first element is a real and positive number. The sum of the squared moduli of the c parameters must be equal to 1/(2*pi).
M	Number of components in the NNTS

Value

The function returns the value of the distribution function at theta

Author(s)

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

Examples

```
cpars<-nntsrandominitial(2)
nntsDistributioninterval0to1(pi, cpars, 2)
```

---

nntsDistributioninterval0to2pi

*NNTS distribution function for data defined in the interval [0,2*pi)*

Description

Computes the distribution function for the data at theta

Usage

```
nntsDistributioninterval0to2pi(theta, cpars = 1/sqrt(2 * pi), M = 0)
```

Arguments

theta	Value at which the distribution function is computed
cpars	Vector of complex numbers of dimension M+1. The first element is a real and positive number. The sum of the SQUARED moduli of the c parameters must be equal to 1/(2*pi).
M	Number of components in the NNTS

Value

The function returns the value of the distribution function at theta

Author(s)

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

Examples

```
cpars<-nntsrandominitial(3)
nntsDistributioninterval0to2pi(0, cpars, 3)
nntsDistributioninterval0to2pi(pi, cpars, 3)
nntsDistributioninterval0to2pi(2*pi-0.00000001, cpars, 3)
nntsDistributioninterval0to2pi(2*pi, cpars, 3)
nntsDistributioninterval0to2pi(3*pi, cpars, 3)
```

---

nntsestimationSymmetric — *NNTS Symmetric Coefficient estimation*

Description

Computes the maximum likelihood estimates of the symmetric NNTS parameters

Usage

```
nntsestimationSymmetric(M = 0, data, maxit = 500)
```

Arguments

`M`	Number of components in the NNTS
`data`	Vector of angles in radians
`maxit`	Maximum number of iterations in the optimization algorithm

Value

`coef`	Vector of length M+1. The first M components are the squared moduli of the c parameters, and the last number is the mean of symmetry
`loglik`	Optimum log-likelihood value
`AIC`	Value of Akaike's Information Criterion
`BIC`	Value of Bayesian Information Criterion
`convergence`	An integer code: zero indicates successful convergence; error codes are the following: one indicates that the iteration limit maxit has been reached, and 10 indicates degeneracy of the Nelder-Mead simplex

Note

For the maximization of the loglikelihood function the function constrOptim from the package stats is used

Author(s)

Juan Jose Fernandez-Duran y Maria Mercedes Gregorio-Dominguez

References

Fernandez-Duran, J.J., Gregorio-Dominguez, M.M. (2009) Symmetric Circular Distributions Based on Nonnegative Trigonometric Sums. Working Paper, DE-C09.12, Department of Statistics, ITAM, Mexico

Examples

```
b<-c(runif(10,3*pi/2,2*pi-0.00000001),runif(10,pi/2,pi-0.00000001))
estS<-nntsestimationSymmetric(2,b)
nntsplotSymmetric(estS$coef,2)
```

---

nntsloglik *NNTS log-likelihood function*

Description

Computes the log-likelihood function with NNTS density for data

Usage

```
nntsloglik(data, cpars = 1/sqrt(2 * pi), M = 0)
```

Arguments

data	Vector with observed angles in radians.
cpars	Vector of complex numbers of dimension M+1. The first element is a real and positive number. The sum of the squared moduli of the c parameters must be equal to 1/(2*pi).
M	Number of components in the NNTS

Value

The function returns the value of the log-likelihood function for the data

Author(s)

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

References

Fernandez-Duran, J.J. (2004). Circular Distributions Based on Nonnegative Trigonometric Sums, Biometrics, 60(2), 499-503.

Examples

```
a<-c(runif(10,3*pi/2,2*pi-0.00000001),runif(10,pi/2,pi-0.00000001))
est<-nntsmanifoldnewtonestimation(a,2)
ccoef<-est$cestimates[,2]
nntsloglik(a,ccoef,2)
```

`nntsloglikInterval0to1` *NNTS log-likelihood function for the incidence data defined in the interval [0,1)*

Description

Computes the log-likelihood function for incidence data (number of observed values in certain intervals defined in the interval [0,1))

Usage

```
nntsloglikInterval0to1(data, cutpoints, cpars = 1/sqrt(2 * pi), M = 0)
```

Arguments

`data`	Number of observations in each interval
`cutpoints`	Vector of size length(data)+1 with the limits of the intervals
`cpars`	Vector of complex numbers of dimension M+1. The first element is a real and positive number. The sum of the SQUARED moduli of the c parameters must be equal to 1/(2*pi).
`M`	Number of components in the NNTS

Value

The function returns the value of the log-likelihood function for data

Author(s)

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

Examples

```
data<-c(1,2,6,4,1)
cutpoints<-c(0,0.2,0.4,0.6,0.8,0.9999999)
cpars<-nntsrandominitial(1)
nntsloglikInterval0to1(data, cutpoints, cpars, 1)
```

nntsloglikInterval0to2pi

*NNTS log-likelihood function for the incidence data defined in the interval [0,2*pi)*

Description

Computes the log-likelihood function for incidence data (number of observed values in certain intervals defined in the interval [0,2*pi))

Usage

```
nntsloglikInterval0to2pi(data, cutpoints, cpars = 1/sqrt(2 * pi), M = 0)
```

Arguments

`data`	Number of observations in each interval
`cutpoints`	Vector of size length(data)+1 with the limits of the exhaustive and mutually exclusive intervals in which the interval [0,2*pi) is divided.
`cpars`	Vector of complex numbers of dimension M+1. The first element is a real and positive number. The sum of the SQUARED moduli of the c parameters must be equal to 1/(2*pi).
`M`	Number of components in the NNTS density

Value

The function returns the value of the log-likelihood function for the data

Author(s)

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

Examples

```
data<-c(2,3,6,4)
cutpoints<-c(0,pi/2,pi,3*pi/2,2*pi-0.00000001)
est<-nntsmanifoldnewtonestimationinterval0to2pi(data,cutpoints,M=1)
cpars<-est$cestimates[,2]
nntsloglikInterval0to2pi(data,cutpoints,cpars,M=1)
```

nntsloglikSymmetric — *NNTS symmetric log-likelihood function*

Description

Computes the log-likelihood function with NNTS symmetric density for the data

Usage

```
nntsloglikSymmetric(cpars = c(0, 0), M = 0, data)
```

Arguments

cpars	Vector of real numbers of dimension M+1. The first M numbers are the squared moduli of the c parameters. The sum must be less than 1/(2*pi). The last number is the mean of symmetry
M	Number of components in the NNTS
data	Vector with angles in radians. The first column is used if data are a matrix

Value

The function returns the value of the log-likelihood function for the data

Note

The default values provide the Uniform circular log-likelihood for the data

Author(s)

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

References

Fernandez-Duran, J.J., Gregorio-Dominguez, M.M. (2009) Symmetric Circular Distributions Based on Nonnegative Trigonometric Sums. Working Paper, DE-C09.12, Department of Statistics, ITAM, Mexico

Examples

```
nntsloglikSymmetric(c(.01,.02,2),2,t(c(pi,pi/2,2*pi,pi)))
```

nntsmanifoldnewtonestimation

Parameter estimation for NNTS distributions

Description

Computes the maximum likelihood estimates of the NNTS parameters, using a Newton algorithm on the hypersphere

Usage

```
nntsmanifoldnewtonestimation(data, M=0, iter=1000, initialpoint = FALSE, cinitial)
```

Arguments

`data`	Vector of angles in radians
`M`	Number of components in the NNTS
`iter`	Number of iterations
`initialpoint`	TRUE if an initial point for the optimization algorithm will be used
`cinitial`	Vector of size M+1. The first element is real and the next M elements are complex (values for c_0 and $c_1, ...,c_M$). The sum of the squared moduli of the parameters must be equal to 1/(2*pi)

Value

`cestimates`	Matrix of (M+1)x2. The first column is the parameter numbers, and the second column is the c parameter's estimators
`loglik`	Optimum log-likelihood value
`AIC`	Value of Akaike's Information Criterion
`BIC`	Value of Bayesian Information Criterion
`gradnormerror`	Gradient error after the last iteration

Author(s)

Juan Jose Fernandez-Duran y Maria Mercedes Gregorio-Dominguez

References

Fernandez-Duran, J.J., Gregorio-Dominguez, M.M. (2010). Maximum Likelihood Estimation of Nonnegative Trigonometric Sums Models by Using a Newton-like Algorithm on Manifolds, Working Paper, Department of Statistics, ITAM, DE-C10.8

Examples

```
set.seed(200)
a<-c(runif(10,3*pi/2,2*pi-0.00000001),runif(10,pi/2,pi-0.00000001))
#Estimation of the NNTSdensity with 2 components for data and 200 iterations
nntsmanifoldnewtonestimation(a,2,iter=200)

data(Turtles_radians)
#Empirical analysis of data
Turtles_hist<-hist(Turtles_radians,breaks=10,freq=FALSE)
#Estimation of the NNTS density with 3 componentes for data
nntsmanifoldnewtonestimation(Turtles_radians,3,iter=200)
```

---

nntsmanifoldnewtonestimationgradientstop

Maximum likelihood estimates of the NNTS parameters

Description

Computes the maximum likelihood estimates of the NNTS parameters, using a Newton algorithm on the hypersphere with the option to specify a minimum value of the norm of the gradient error to stop the algorithm

Usage

```
nntsmanifoldnewtonestimationgradientstop(data, M = 0, iter = 1000, initialpoint = FALSE,
cinitial,gradientstop=1e-10)
```

Arguments

`data`	Vector of angles in radians
`M`	Number of components in the NNTS
`iter`	Number of iterations
`initialpoint`	TRUE if an initial point for the optimization algorithm will be used
`cinitial`	Vector of size M+1. The first element is real and the next M elements are complex (values for c_0 and $c_1, \ldots, c_M$). The sum of the squared moduli of the parameters must be equal to $\frac{1}{7}2\pi$
`gradientstop`	The value of the norm of the gradient error of the Newton algorithm at which the algorithms stops

Value

`cestimates`	Matrix of (M+1)x2. The first column is the parameter numbers, and the second column is the c parameter???s estimators
`loglik`	Optimum log-likelihood value
`AIC`	Value of Akaike???s Information Criterion
`BIC`	Value of Bayesian Information Criterion
`gradnormerror`	Gradient error after the last iteration

Author(s)

Juan Jose Fernandez-Duran y Maria Mercedes Gregorio-Dominguez

References

Fernandez-Duran, J.J., Gregorio-Dominguez, M.M. (2010). Maximum Likelihood Estimation of Nonnegative Trigonometric Sums Models by Using a Newton-like Algorithm on Manifolds, Working Paper, Department of Statistics, ITAM, DE-C10.8

Examples

```
set.seed(200)
a<-c(runif(10,3*pi/2,2*pi-0.00000001),runif(10,pi/2,pi-0.00000001))
#Estimation of the NNTSdensity with 2 components for data and gradientstop at 1e-12
nntsmanifoldnewtonestimationgradientstop(a,2,gradientstop=1e-12)
data(Turtles_radians)
#Empirical analysis of data
Turtles_hist<-hist(Turtles_radians,breaks=10,freq=FALSE)
#Estimation of the NNTS density with 3 componentes for data and gradientstop at 1e-12
nntsmanifoldnewtonestimationgradientstop(Turtles_radians,3,gradientstop=1e-12)
```

---

nntsmanifoldnewtonestimationinterval0to1 *Parameter estimation for grouped data defined in [0,1)*

Description

Parameter estimation for incidence data (number of observed values in certain intervals defined over [0,1))

Usage

```
nntsmanifoldnewtonestimationinterval0to1(data, cutpoints, subintervals, M = 0, iter=1000,
initialpoint = FALSE, cinitial)
```

Arguments

`data`	Frequency of data on each interval
`cutpoints`	Vector with the limits of intervals. The length of cutpoints must be one plus the length of the data
`subintervals`	Number of intervals
`M`	Number of components in the NNTS
`iter`	Number of iterations
`initialpoint`	TRUE if an initial point for the optimization algorithm will be used
`cinitial`	Vector of size M+1. The first element is real and the next M elements are complex (values for c_0 and $c_1, ...,c_M$).The sum of the squared moduli of the parameters must be equal to 1/(2*pi)

Value

`cestimates`	Matrix of M+1 * 2. The first column is the parameter numbers and the second column is the c parameter's estimators
`loglik`	Optimum loglikelihood value
`AIC`	Value of Akaike's Information Criterion
`BIC`	Value of Bayesian Information Criterion
`gradnormerror`	Gradient error after the last iteration

Author(s)

Juan Jose Fernandez-Duran y Maria Mercedes Gregorio-Dominguez

Examples

```
data<-c(1,2,4,6,1)
cutpoints<-c(0,0.2,0.4,0.6,0.8,0.999999999)
nntsmanifoldnewtonestimationinterval0to1(data, cutpoints, length(data), 1)
```

`nntsmanifoldnewtonestimationinterval0to2pi` *Parameter estimation for grouped data defined in [0,2*pi)*

Description

Parameter estimation for incidence data (number of observed values in certain intervals defined over [0,2*pi))

Usage

```
nntsmanifoldnewtonestimationinterval0to2pi(data, cutpoints,
subintervals,M = 0, iter=1000, initialpoint = FALSE, cinitial)
```

Arguments

`data`	Frequency of data on each interval
`cutpoints`	Vector with the limits of intervals. The length of cutpoints has to be one plus the length of the data
`subintervals`	Number of intervals
`M`	Number of components in the NNTS
`iter`	Number of iterations
`initialpoint`	TRUE if an initial point for the optimization algorithm will be used
`cinitial`	A vector of size M+1. The first element is real, and the next M elements are complex (values for \$c_0\$ and \$c_1, ... ,c_M\$).The sum of the squared moduli of the parameters must be equal to 1/(2*pi)

Value

cestimates	Matrix of M+1 * 2. The first column is the parameter numbers, and the second column is the c parameter's estimators
loglik	Optimum log-likelihood value
AIC	Value of Akaike's Information Criterion
BIC	Value of Bayesian Information Criterion
gradnormerror	Gradient error after last iteration

Author(s)

Juan Jose Fernandez-Duran y Maria Mercedes Gregorio-Dominguez

Examples

```
data<-c(1,2,6,4)
cutpoints<-c(0,pi/2,pi,3*pi/2,2*pi-0.00000001)
nntsmanifoldnewtonestimationinterval0to2pi(data, cutpoints, length(data),1)
```

---

nntsplot *Plots the NNTS density*

Description

Plots the NNTS density

Usage

```
nntsplot(cpars = 1/sqrt(2 * pi), M = 0, ...)
```

Arguments

cpars	Vector of complex numbers of dimension M+1. The first element is a real and positive number. The sum of the SQUARED moduli of the c parameters must be equal to 1/(2*pi).
M	Number of components in the NNTS
...	Arguments passed to the function curve

Examples

```
data(Turtles_radians)
#Empirical analysis of data
Turtles_hist<-hist(Turtles_radians,breaks=10,freq=FALSE)
#Estimation of the NNTS density with 3 componentes for data
est<-nntsmanifoldnewtonestimation(Turtles_radians,3,iter=200)
est
#plot the histogram
```

```
plot(Turtles_hist, freq=FALSE)
#add the estimated density to the histogram
nntsplot(est$cestimates[,2],3,add= TRUE)
```

---

nntsplotInterval0to1 *Plots an NNTS density for a variable defined in the interval [0,1)*

Description

Plots the NNTS density for a variable defined in the interval [0,1)

Usage

```
nntsplotInterval0to1(cpars = 1/sqrt(2 * pi), M = 0, ...)
```

Arguments

cpars	Vector of complex numbers of dimension M+1. The first element is a real and positive number. The sum of the SQUARED moduli of the c parameters must be equal to 1/(2*pi).
M	Number of components in the NNTS
...	Arguments passed to the function curve

Author(s)

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

Examples

```
data<-c(1,2,4,6,2)
cutpoints<-c(0,0.2,0.4,0.6,0.8,0.9999999)
est<-nntsmanifoldnewtonestimationinterval0to1(data,cutpoints,5,1)
cpars<-est$cestimates[,2]
nntsplotInterval0to1(cpars, 1)
```

---

nntsplotSymmetric *Plots a symmetric NNTS density function*

Description

Plots the Symmetric NNTS density function

Usage

```
nntsplotSymmetric(cpars = c(0, 0), M = 0, ...)
```

Arguments

`cpars`	Vector of real numbers of dimension 2M. The first 2M-1 numbers are the squared moduli of the c parameters. The sum must be less than 1/(2*pi). The last number is the mean of symmetry
`M`	Number of components in the NNTS
`...`	Arguments passed to the function curve

Author(s)

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

References

Fernandez-Duran, J.J., Gregorio-Dominguez, M.M. (2009) Symmetric Circular Distributions Based on Nonnegative Trigonometric Sums. Working Paper, DE-C09.12, Department of Statistics, ITAM, Mexico

---

`nntsrandominitial` *Initial random point*

Description

This function generates a random point on the surface of the (M+1)-dimensional unit hypersphere

Usage

```
nntsrandominitial(M=1)
```

Arguments

`M`	Number of components in the NNTS

Value

Returns a valid initial point for the estimation functions

Author(s)

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

Examples

```
nntsrandominitial(3)
nntsrandominitial(7)
```

`nntsrandominitialSymmetric` *Initial random point*

Description

This function generates a random point on the surface of the (M+1)-dimensional unit hypersphere

Usage

```
nntsrandominitialSymmetric(M)
```

Arguments

`M`	Number of components in the NNTS

Value

Returns a valid initial point for the estimation functions nntsestimation and nntsestimationSymmetric

Author(s)

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

`nntssimulation` *NNTS density simulation function*

Description

Simulation for the density function for the NNTS model

Usage

```
nntssimulation(nsim=1, cpars = 1/(2 * pi), M = 0)
```

Arguments

`nsim`	Number of simulations
`cpars`	Parameters of the model. A vector of complex numbers of dimension M+1. The sum of the squared moduli of the c parameters must be equal to 1/(2*pi).
`M`	Number of components in the NNTS model

Value

simulations	The function generates nsim random values from the MNNTS density function
conteo	Number of uniform random numbers used for simulations

Author(s)

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

Examples

```
M<-3
ccoef<-nntsrandominitial(M)
data<-nntssimulation(10,ccoef,M)
data
```

---

nntsSymmetricDensity *Symmetric NNTS density function*

Description

Density function for the Symmetric NNTS

Usage

```
nntsSymmetricDensity(cpars = c(0, 0), M = 0, theta)
```

Arguments

cpars	Vector of real numbers of dimension 2*M. The first M numbers are the squared moduli of the c parameters. The sum must be less than 1/(2*pi). The last number is the mean of symmetry
M	Number of components in the NNTS
theta	Angle in radians

Value

The function returns the density function evaluated at theta

Note

The default values provide the uniform circular density

Author(s)

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

References

Fernandez-Duran, J.J., Gregorio-Dominguez, M.M. (2009) Symmetric Circular Distributions Based on Nonnegative Trigonometric Sums. Working Paper, DE-C09.12, Department of Statistics, ITAM, Mexico

---

nntsuniformitytestlikelihoodratio

Computes the statistic and critical values of the circular uniformity test

---

Description

Computes the statistic and critical values at 10%, 5% and 1% of the circular uniformity test based on the NNTS likelihood ratio for M values from 1 to 7 and any sample size.

Usage

```
nntsuniformitytestlikelihoodratio(data,M=1, iter=1000, initialpoint = FALSE,
cinitial,gradientstop=1e-10)
```

Arguments

`data`	Vector of angles in radians
`M`	Number of components in the NNTS
`iter`	Number of iterations
`initialpoint`	TRUE if an initial point for the optimization algorithm will be used
`cinitial`	Vector of size M+1. The first element is real and the next M elements are complex (values for c_0 and $c_1, \ldots, c_M$). The sum of the squared moduli of the parameters must be equal to $\frac{1}{2\pi}$
.	
`gradientstop`	The value of the gradient of the Newton algorithm at which the algorithms stops

Value

`gradient`	Gradient error after the last iteration
`likratiounifstat`	Value of the likelihood ratio NNTS circular uniformity test statistic
`criticalvalue10percent`	Critical value at a 10% significance level of the likelihood ratio NNTS circular uniformity test
`criticalvalue05percent`	Critical value at a 5% significance level of the likelihood ratio NNTS circular uniformity test
`criticalvalue01percent`	Critical value at a 1% significance level of the likelihood ratio NNTS circular uniformity test

Author(s)

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

References

Fernandez-Duran and J. J. and Gregorio-Dominguez and M. M (2022). Sums of Independent Circular Random Variables and Maximum Likelihood Circular Uniformity Tests Based on Nonnegative Trigonometric Sums Distributions, arXiv preprint arXiv:2212.01416

Examples

```
set.seed(200)
a<-2*pi*runif(50)
#NNTS likelihood ratio circular uniformity test for M=2 and gradientstop at 1e-09
nntsuniformitytestlikelihoodratio(data=a,M=2,gradientstop=1e-09)
data(Turtles_radians)
#NNTS likelihood ratio circular uniformity test for M=5 and gradientstop at 1e-12
nntsuniformitytestlikelihoodratio(data=Turtles_radians,M=5,gradientstop=1e-09)
```

ProteinsAAA *Dihedral angles in protein*

Description

Dataset of the dihedral angles in a protein between three consecutive Alanine (Ala) amino acids. This dataset was constructed from the recommended July 2003 list of proteins via the algorithm in Hobohm et al. (1992). This algorithm selects a representative sample of proteins from the vast Protein Data Bank (PDB, Berman et al., 2000). The dataset contains 233 pairs of dihedral angles.

Usage

```
data(ProteinsAAA)
```

Format

Two columns of angles in radians

Source

Protein Data Bank (PDB)

References

Hobohm, U. and Scharf, M. and Schneider, R. and Sander, C. (1992) Selection of a Representative Set of Structures from the Brookhaven Protein Data Bank, Protein Science, 1, 409-417. Berman, H. M. and Westbrook, J. and Feng, Z. and Gilliand, G. and Bhat, T. N. and Weissing, H. and Shyndialov, I. N. and Bourne, P. E. (2000) The Protein Data Bank, Nucleic Acids Research, 28, 235-242.

snntsdensity *SNNTS density function for spherical data*

Description

Density function for the SNNTS model for spherical data

Usage

```
snntsdensity(data, cpars = 1, M = c(0,0))
```

Arguments

data	Matrix of angles in radians. The first column contains longitude data (between zero and 2*pi), and second column contains latitude data (between zero and pi), with one row for each data point
cpars	Vector of complex numbers of dimension prod(M+1). The first element is a real and positive number. The sum of the squared moduli of the c parameters must be equal to one.
M	Vector with the number of components in the SNNTS for each dimension

Value

The function returns the density function evaluated for each row in the data

Note

The parameters cinitial and cestimates used by this function are the transformed parameters of the SNNTS density function, which lie on the surface of the unit hypersphere

Author(s)

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

References

Juan Jose Fernandez-Duran, Maria Mercedes Gregorio-Dominguez (2016). CircNNTSR: An R Package for the Statistical Analysis of Circular, Multivariate Circular, and Spherical Data Using Nonnegative Trigonometric Sums. Journal of Statistical Software, 70(6), 1-19. doi:10.18637/jss.v070.i06

Examples

```
data(Datab6fisher_ready)
data<-Datab6fisher_ready
M<-c(2,3)
cpars<-rnorm(prod(M+1))+rnorm(prod(M+1))*complex(real=0,imaginary=1)
cpars[1]<-Re(cpars[1])
cpars<- cpars/sqrt(sum(Mod(cpars)^2))
```

```
snntsdensity(data, cpars, M)
```

`snntsdensityplot` *Plots a SNNTS density for spherical data*

Description

Computes the points needed to plot the SNNTS density function for spherical data

Usage

```
snntsdensityplot(long, lat, cpars = 1, M = c(0,0))
```

Arguments

`long`	Grid for longitude. Vector with values between zero and 2*pi
`lat`	Grid for latitude. Vector with values between zero and pi
`cpars`	Vector of complex numbers of dimension prod(M+1). The sum of the squared moduli of the c parameters must be equal to one
`M`	Vector with the number of components in the SNNTS for each dimension

Value

The points needed to plot the SNNTS density function

Note

The parameters cpars used by this function are the transformed parameters of the SNNTS density function, which lie on the surface of the unit hypersphere

Author(s)

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

References

Fernandez-Duran J. J. y Gregorio Dominguez, M. M. (2008) Spherical Distributions Based on Non-negative Trigonometric Sums, Working Paper, Statistics Department, ITAM, DE-C08.6

Examples

```
set.seed(200)
data(Datab6fisher_ready)
data<-Datab6fisher_ready
M<-c(4,4)
cest<-snntsmanifoldnewtonestimation(data, M, iter=150)
cpars<-cest$cestimates[,3]
longitud<-seq(0,360,10)*(pi/180)
latitud<-seq(0,180,5)*(pi/180)
z<-outer(longitud,latitud,FUN="snntsdensityplot",cpars,M)
persp(longitud,latitud,z,theta=45,phi=30)
contour(longitud,latitud,z)
points(data[,1],data[,2])
```

---

snntsloglik *SNNTS log-likelihood function for spherical data*

Description

Computes the log-likelihood function with SNNTS density for spherical data

Usage

```
snntsloglik(data, cpars = 1, M = c(0,0))
```

Arguments

data	Matrix of angles in radians. The first column contains longitude data (between zero and 2*pi), and the second column containslatitude data (between zero and pi), with one row for each data point
cpars	Vector of complex numbers of dimension prod(M+1). The first element is a real and positive number. The first M[1]+1 elements correspond to longitude, the next M[2]+1 elements correspond to latitude. The sum of the squared moduli of the c parameters must be equal to 1
M	Vector with number of components in the SNNTS for each dimension

Value

The function returns the value of the log-likelihood function for the data

Note

The parameters cpars used by this function are the transformed parameters of the SNNTS density function, which lie on the surface of the unit hypersphere

Author(s)

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

References

Fernandez-Duran J. J. y Gregorio Dominguez, M. M. (2008) Spherical Distributions Based on Non-negative Trigonometric Sums, Working Paper, Statistics Department, ITAM, DE-C08.6

Examples

```
data(Datab6fisher_ready)
data<-Datab6fisher_ready
M<-c(4,4)
cpars<-rnorm(prod(M+1))+rnorm(prod(M+1))*complex(real=0,imaginary=1)
cpars[1]<-Re(cpars[1])
cpars<- cpars/sqrt(sum(Mod(cpars)^2))
snntsdensity(data, cpars, M)
snntsloglik(data, cpars, M)
```

---

snntsmanifoldnewtonestimation

Parameter estimation for SNNTS distributions for spherical data

---

Description

Computes the maximum likelihood estimates of the SNNTS model parameters using a Newton algorithm on the hypersphere

Usage

```
snntsmanifoldnewtonestimation(data, M = c(0,0), iter = 1000,
initialpoint = FALSE, cinitial)
```

Arguments

`data`	Matrix of angles in radians, with one row for each data point. The first column contains longitude data (between zero and 2*pi), and second column contains latitude data (between zero and pi), with one row for each data point
`M`	Vector with number of components in the SNNTS for each dimension
`iter`	Number of iterations
`initialpoint`	TRUE if an initial point for the optimization algorithm will be used
`cinitial`	Initial value for cpars for the optimization algorithm, avector of complex numbers of dimension prod(M+1). The first element is a real and positive number. The sum of the squared moduli of the c parameters must be equal to one.

Value

cestimates	Matrix of prod(M+1)*(3). The first two columns are the parameter numbers, and the last column is the c parameter's estimators
loglik	Optimum log-likelihood value
AIC	Value of Akaike's Information Criterion
BIC	Value of Bayesian Information Criterion
gradnormerror	Gradient error after the last iteration

Note

The parameters cinitial and cestimates used by this function are the transformed parameters of the SNNTS density function, which lie on the surface of the unit hypersphere

Author(s)

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

References

Fernandez-Duran J. J. y Gregorio Dominguez, M. M. (2008) Spherical Distributions Based on Nonnegative Trigonometric Sums, Working Paper, Statistics Department, ITAM, DE-C08.6

Examples

```
set.seed(200)
data(Datab6fisher_ready)
data<-Datab6fisher_ready

M<-c(4,4)
cpar<-rnorm(prod(M+1))+rnorm(prod(M+1))*complex(real=0,imaginary=1)
cpar[1]<-Re(cpar[1])
cpar<- cpar/sqrt(sum(Mod(cpar)^2))

cest<-snntsmanifoldnewtonestimation(data,c(4,4),100,TRUE,cpar)
cest
cest<-snntsmanifoldnewtonestimation(data,c(1,2),100)
cest
```

---

snntsmarginallatitude *Marginal density function for latitude of the SNNTS model for spherical data*

Description

Marginal density function for latitude of the SNNTS model for spherical data

Usage

```
snntsmarginallatitude(data, cpars = 1, M = c(0,0))
```

Arguments

`data`	Vector of angles in radians, with one row for each data point. The data must be between zero and pi.
`cpars`	Vector of complex numbers of dimension prod(M+1). The first element is a real and positive number. The sum of the squared moduli of the c parameters must be equal to one
`M`	Vector with the number of components in the SNNTS for each dimension

Value

The function returns the SNNTS marginal density function for latitude evaluated at data

Note

The parameters cpars used by this function are the transformed parameters of the SNNTS density function, which lie on the surface of the unit hypersphere

Author(s)

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

References

Fernandez-Duran J. J. y Gregorio Dominguez, M. M. (2008) Spherical Distributions Based on Nonnegative Trigonometric Sums, Working Paper, Statistics Department, ITAM, DE-C08.6

Examples

```
set.seed(200)
data(Datab6fisher_ready)
data<-Datab6fisher_ready
M<-c(1,2)
cest<-snntsmanifoldnewtonestimation(data, M,iter=150)
lat<-snntsmarginallatitude(seq(0,pi,.1),cest$cestimates[,3],M)
plot(seq(0,pi,.1),lat,type="l")
```

`snntsmarginallongitude` *Marginal density function for the longitude of the SNNTS model for spherical data*

Description

Marginal density function for the longitude of the SNNTS model for spherical data

Usage

```
snntsmarginallongitude(data, cpars = 1, M = c(0,0))
```

Arguments

`data`	Vector of angles in radians, with one row for each data point. The data must be between zero and 2*pi
`cpars`	Vector of complex numbers of dimension prod(M+1). The first element is a real and positive number. The first M[1]+1 elements correspond to longitude, and the next M[2]+1 elements correspond to latitude. The sum of the squared moduli of the c parameters must be equal to one.
`M`	Vector with number of components in the SNNTS for each dimension

Value

The function returns the density function evaluated for the data

Note

The parameters cpars used by this function are the transformed parameters of the SNNTS density function, which lie on the surface of the unit hypersphere

Author(s)

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

References

Fernandez-Duran J. J. y Gregorio Dominguez, M. M. (2008) Spherical Distributions Based on Non-negative Trigonometric Sums, Working Paper, Statistics Department, ITAM, DE-C08.6

Examples

```
set.seed(200)
data(Datab6fisher_ready)
data<-Datab6fisher_ready
M<-c(1,2)
cest<-snntsmanifoldnewtonestimation(data, M,iter=150)
long<-snntsmarginallongitude(seq(0,2*pi,.1),cest$cestimates[,3],M)
plot(seq(0,2*pi,.1),long,type="l")
```

---

snntssimulation *SNNTS density simulation function*

Description

Simulation for the density function for the SNNTS model

Usage

```
snntssimulation(nsim=1, cpars =(1/(2*pi))^2, M = c(0,0))
```

Arguments

nsim	Number of simulations
cpars	Vector of complex numbers of dimension prod(M+1). The first element is a real and positive number. The first M[1]+1 elements correspond to longitude, the next M[2]+1 elements correspond to latitude. The sum of the squared moduli of the c parameters must be equal to one
M	Vector with the number of components in the SNNTS for each dimension

Value

simulations	The function generates nsim random values from the SNNTS density function
conteo	Number of uniform random numbers used for simulations

Author(s)

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

Examples

```
M<-c(2,3)
R<-length(M)
ccoef<-mnntsrandominitial(M,R)
data<-mnntssimulation(10,ccoef,M,R)
data
```

`SuicidesMexico2005` *Suicides in Mexico during 2005*

Description

Monthly number of suicides in Mexico during 2005

Usage

```
data(SuicidesMexico2005)
```

Format

Integer values

Source

INEGI (Mexican National Statistical Agency) www.inegi.gob.mx

`Turtles` *Movements of turtles*

Description

Data measurement of the directions taken by 76 turtles after treatment

Usage

```
data(Turtles)
```

Format

Directions of turtles in degrees

Source

Stephens (1969) Techniques for directional data. Technical Report 150. Dept. of Statistics, Stanford University. Stanford, CA.

References

N.I. Fisher (1993) Statistical analysis of circular data. Cambridge University Press.

Turtles_radians *Movements of turtles*

Description

Data measurement of the directions taken by 76 turtles after treatment

Usage

```
data(Turtles_radians)
```

Format

Directions of turtles in radians

Source

Stephens (1969) Techniques for directional data. Technical Report 150. Dept. of Statistics, Stanford University. Stanford, CA.

References

N.I. Fisher (1993) Statistical analysis of circular data. Cambridge University Press.

WindDirectionsTrivariate *Wind directions*

Description

Wind directions registered at the monitoring stations of San Agustin located in the north, Pedregal in the southwest, and Hangares in the southeast of the Mexico Central Valley's at 14:00 on days between January 1, 1993 and February 29, 2000. There are a total of 1,682 observations

Usage

```
data(WindDirectionsTrivariate)
```

Format

Three columns of angles in radians

Source

Mexico Central Valleys pollution monitoring network. RAMA SIMAT (Red Automatica de Monitoreo Ambiental)

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